Some Descriptional Complexity Problems in Finite Automata Theory *

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ABSTRACT

Automata Theory is one of the oldest and well studied areas in theoretical computer science. Almost all not very hard problems, yet of main interest have been solved. The exciting developments in theoretical computer science for more than two decades ago produced very specialized computer scientists with few knowledge about other areas and much less about the context of fundamental theory.

In this paper, we discuss three problems pertaining to descriptional complexity of finite automata. The long standing problem of estimating the size of the minimal nondeterministic finite automaton $ns(L)$ for a regular language L is considered. We present the best so far lower bound for $ns(L)$ for some regular languages L. Some issues related to ambiguity on nondeterministic finite automaton are also discussed. Finally, we discussed the fundamental problem since 1978 about the relationship between 2-way deterministic finite automaton and 2-way nondeterministic finite automaton.

We argue that automata theory has enough basic questions to be resolved. If one wants to put automata theory in the place where it was once in computer science research, one must not only look at applications of the theory but consider also solving problems of great impact.

Keywords

Descriptional Complexity, Finite Automata, Regular Languages, Lower Bounds

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1. INTRODUCTION

Finite automaton is one of the oldest and well studied computation models in theoretical computer science. The basic model called the deterministic finite automaton (dfa) was introduced independently in [22, 24, 30] while the so-called nondeterministic finite automaton (nfa) was defined in [26] during the 50's. This models have been the basis of studies concerning the fundamental questions on computation and complexity.

For the past 20 years the interest of the computing community become less and less in theory in particular, in automata theory. Although important conferences in theory, like STACS, ICALP, STOC and FOCS are still publishing new results in theoretical computer science relatively few papers deals with automata theory. Some colleagues believed that the renaissance of automata theory is possible by looking at its applications [33].

In particular, Sheng Yu [33] mentioned that aside from the fact that many funding agencies are becoming reluctant in supporting research projects in automata theory, the subject area is not anymore considered to be in the main-stream and is regarded as out of fashion and even of no use.

While looking for applications of automata theory is quite nice and probably interesting, we share the belief that applications although important is not enough for automata theory to regain its lost position in computer science [10]. Realized that because of the exciting theoretical developments for more than two decades ago in the subject, todays computer scientists are so specialized. These people know very little about other areas and even less about the whole context of fundamental theory.

Moreover, researchers tend to solve problems in areas where there are enough interesting not too hard problems because they are required to publish. Some write the 10th edition of the 201st variation of a problem as it was described by Hromkovič in [10]. This is to say that almost all not too hard problems of main interest are solved in the theory of automata and we are left with really very hard ones.

It is clear that we can only hope to be in the 10-20 main stream research in computer science. However, realized that specialized conferences and workshops e.g. DLT and DCAGRS have been established in support of revitalizing interest in researches in theoretical computer science, in particular automata and formal language theory.

The main goal of this paper is to present some interesting problems which are fundamental to the theory of computations. This is to say that we have enough basic research problem of above mentioned importance. We mentioned only here some problems, in particular those that concerns descriptional complexity of finite automata.

The paper is organized as follows; Section 2 provides a discussion on results in finding minimal nondeterministic finite automaton. Section 3 considers ambiguity of nondeterministic finite automaton. Section 4 discusses fundamental problem concerning determinism and nondeterminism.

2. MINIMAL NFA FOR A REGULAR L

We are all aware that finite state automaton was introduced independently by various individuals as early as the 50's [22, 24, 30]. Nondeterminism on this model was introduced in [26]. However, it was subsequently shown in [26] that the computational power of the basic model and the nondeterministic one are equivalent through subset construction technique.

This resulted to answering question regarding the relative power of nondeterminism versus determinism to the level of descriptional complexity. That is, we compare the cardinality of the finite set of states of these two machines. Also minimizing the number of required finite set of states of an automaton accepting set of strings is still an issue to be resolved, especially for nfa's.

One of the principal open problems in automata theory is:

Problem 1: *Estimate the size of the minimal nondeterministic finite automaton for a regular language L.*

At the moment we are not aware of a method that would at least assure an approximation of this value. This is contrary to the known result in the deterministic case. The minimum dfa for a regular language L can be obtained efficiently in $O(n \log n)$, where n is the number of states of the dfa [8].

The problem of finding the size of the minimal nfa for a regular language L is known to be PSPACE-complete except when the alphabet is a singleton set [6]. The minimal nfa for a regular language L is not necessarily unique. The search for the general procedure is not likely to be successful because of its PSPACE-completeness [6, 17].

Thus the investigation points to the development of methods for proving lower bound on $ns(L)$. In [5, 11, 12] communication complexity method was introduced for providing lower bounds on the size of minimal nondeterministic finite automaton. A special form of communication complexity method called fooling set method was used by Glaister and Shallit in [7].

In [1, 2], we introduce a model of computation that captures the uniform computation of automata. This is actually a reasonable generalization of the 2-party model proposed in [15].

Let $L \subseteq \{0,1\}^*$ be a regular language which is to be recognized by k linearly connected independent computers. Every computers C_i ($1 \le i \le k$), gets part α_i ($0 \le |\alpha_i| \le |w|$) of the whole input $w \in \Sigma^*$ s.t. $w = \alpha_1\alpha_2 \ldots \alpha_k$. A **$k$-partition** of w is denoted by by a k-tuple $(\alpha_1, \alpha_2, \ldots, \alpha_k)$.

A **1-way uniform k-party nondeterministic protocol over Σ** is a triple $P_k = \langle \Phi, \{\phi_i\}_{i=2}^{k-1}, \varphi \rangle$ where

- $\Phi \colon \Sigma^* \longrightarrow 2^{\{0,1\}^*}$ is a function.
- $\phi_i \colon \Sigma^* \times \{0,1\}^* \longrightarrow 2^{\{0,1\}^*}$ are functions.
- $\varphi \colon \Sigma^* \times \{0,1\}^* \longrightarrow \{\texttt{accept}, \texttt{reject}\}$ is a function.

Intuitively, P_k computes on any $w \in \Sigma^*$ always w.r.t. a k-partition of w by passing messages from the left to right, beginning from the first computer C_1 consecutively until the last computer C_k. We designate the last computer C_k to be the only one to decide on the acceptance of the input. The message to be transmitted by every C_i, $1 \le i \le k-1$, depends on α_i and the message obtained from C_{i-1}.

A *computation* C of P_k on an input $w \in \Sigma^*$ w.r.t. a k-partition $(\alpha_1, \alpha_2, \ldots, \alpha_k)$ is denoted by a function

$$C \colon \Sigma^* \to 2^{\{0,1\}^*} \cup \{\texttt{accept}, \texttt{reject}\} \quad \text{s.t.} \quad \forall w \in \Sigma^*,$$

$$m_1\$m_2\$\cdots\$m_{k-1}\$m_f \in C(w), \quad \text{where}$$

- $m_1 \in \Phi(\alpha_1)$.
- $m_i \in \phi_i(\alpha_i, m_{i-1})$, $2 \le i \le k-1$.
- $m_f = \varphi(\alpha_k, m_{k-1}) \in \{\texttt{accept}, \texttt{reject}\}$.

The set of all messages that a P_k uses in the computation is given by

$$M(P_k) = \Phi(\Sigma^*) \cup \bigcup_{i=2}^{k-1} \phi_i(\Sigma^* \times \{0,1\}^*) \quad \text{s.t.}$$

- $|M(P_k)| < \infty$
- $M(P_k) \cap M(P_k) \cdot \Sigma^+ = \emptyset$
  or all messages must be *prefix-free.*

If C yields to $m_f = \texttt{accept}$, then C is an **accepting computation** for w. Otherwise, it is **rejecting.** A language L is **accepted** by P_k, i.e. $L = L(P_k)$ iff for all $w \in L$ there is a computation of P_k which yields to $\texttt{accept}$ w.r.t. all k-partitions of w and for all $w \notin L$, no accepting computation may exists.

We define

$$\mathbf{nmc}_k(P_k) = |M(P_k)|$$

as the **1-way uniform k-party message complexity** of P_k. The minimum of all these message complexities for a fixed language is the **1-way uniform k-party nondeter-**

ministic message complexity of L, i.e.

$$\mathbf{nmc}_k(L) = \min\{|M(P_k)| \mid L = L(P_k)\}.$$

In [1], we showed that the k-party nondeterministic message complexity provides a lower bound for $ns(L)$ for any regular language L.

The proof lies on the fact that each participating computer simulates the action of the minimal nfa for any regular language L. As soon as any of the participating computers received an information about the last state visited by the preceding computer while reading the part of the input it has, the receiving one will start simulating the nfa with its part of the input starting from the received state. Realized that the number of messages that will be use is bounded above by $ns(L)$.

Furthermore, in [2], it was shown that for all $k \geq 3$, there is a sequence of regular languages $L_{(k,n)}$ such that

$$ns(L_{(k,n)}) \geq 2^{\Omega(\sqrt{nmc_{(k-1)}(L_{(k,n)})})},$$

where

$$L_{(k,n)} = \{\ w \in \{0,1\}^{kn} \ \mid\ \exists v \leq w. \quad v = xyz \in \{0,1\}^3 \wedge (x = y \vee x \neq z)\,\}.$$

We note that this reasonable generalization of the 2-party model in [15] improved the lower bounding capability of communication complexity vis-a-vis the problem met in [16, 18]. Noticed that this technique may lead to an exponential gap between $ns(L)$ and the provided lower bound for some languages. At the moment this lower bound techniques is the best known so far. However, Problem 1 remains open. Moreover, we must;

Problem 2 *Find a lower bound method for $ns(L)$ that gives a lower bound which is polynomially related to $ns(L)$.*

3. AMBIGUITY ON NFA

Degree of ambiguity is also an intensively investigated concept in automata theory. Measuring the degree of nondeterminism in finite automata had been considered in [16, 21, 25, 27, 32]. To understand the influence of the degree of ambiguity on the size of nfa's is the central question in this sub-area.

Let $\mathcal{M} = (Q, \Sigma, I, \Delta, F)$ be an nfa. We associate the following **ambiguity function:**

$$\text{ambig}_{\mathcal{M}} : \Sigma^* \cup N \longrightarrow N,$$

such that for all $w \in \Sigma^*$,

$$\text{ambig}_{\mathcal{M}}(w) = |\{\, u \in \text{Path}(\mathcal{M}) \mid \chi_\Sigma(u) = w\,\}|$$

and for all $n \in N$, where

$$\text{Path}(\mathcal{M}) = \{\, x \in Q \cdot (\Sigma \cdot Q)^* \mid \exists\, w \in \Sigma^* \text{ s.t. } \chi_\Sigma(x) = w\,\}$$

and

$$\chi_\Sigma : Q \cdot (\Sigma \cup Q)^* \longrightarrow \Sigma^*$$

s.t. for all $q \in Q$, $\chi_\Sigma(q) = \lambda$ while for all $x \in \Sigma$, $\chi_\Sigma(x) = x$.

And we define

$$\text{ambig}_{\mathcal{M}}(n) = \max_{w \in \Sigma^*;\, |w| \leq n} \text{ambig}_{\mathcal{M}}(w).$$

It is imperative that, the above function measures the amount of paths which can be traced successfully from an element of I and ending with a state in F by an automaton on a given input of length at most n.

Ambiguity in nfa's provides information on the number of ways the computing model accepts an input word. If an nfa $\mathcal{M}$ accepts every word in $L(\mathcal{M})$ in one and only one way or computation, then such an nfa is called *unambiguous nfa.* If it accepts words in k computation for some constant k, it is called *constantly ambiguous nfa.* If it accepts the words polynomially many times, then it is a *polynomially ambiguous nfa.* Bounding its asymtotic behavior provides us the following classes of nondeterministic finite automata. We denote by **NFA** the class of all nfa's.

$$\begin{array}{lcl}
\mathbf{DFA} & = & \{\mathcal{M} \in \mathbf{NFA} \mid \forall q \in Q, \sigma \in \Sigma, \\
 & & \quad |(\{(q,\sigma)\} \times Q) \cap \Delta| \leq 1, |I| \leq 1\} \\
\mathbf{UNFA} & = & \{\mathcal{M} \in \mathbf{NFA} \mid \text{ambig}_{\mathcal{M}}(n) \leq 1,\ \forall n \in N\} \\
\mathbf{CAFA} & = & \{\mathcal{M} \in \mathbf{NFA} \mid \text{ambig}_{\mathcal{M}} \in O(1)\,\} \\
\mathbf{PAFA} & = & \{\mathcal{M} \in \mathbf{NFA} \mid \text{ambig}_{\mathcal{M}} \in \cup_{k \in N} O(n^k)\,\}.
\end{array}$$

The following is the ambiguity hierarchy of finite automata:

$$\mathbf{DFA} \subset \mathbf{UNFA} \subset \mathbf{CAFA} \subset \mathbf{PAFA} \subset \mathbf{NFA}.$$

In [16], the following hypothesis on the computation tree of any unambiguous nondeterministic finite automata was given:

> **Strict Tree Property**
> *The computation tree of any minimal $\mathcal{M} \in$ **UNFA** on an input w has exactly one path P from root to a leaf with several nondeterministic guesses and all paths having only one vertex in common with P do not contain any nondeterministic branching.*

However, in [3] we provided a counter example by showing a minimal unambiguous finite automaton that accepts a word wherein the computation tree disobeys the Strict Tree Property. In particular, the automaton $\mathcal{M}$ that would accept the following language,

$$L(\mathcal{M}) = \{\,0,1\,\}^* 1 \cup \{\,0,1\,\}^* 10^+,$$

where

$\mathcal{M} = (Q, \Sigma, \{q_0\}, \Delta, F) \in$ **UNFA** s.t.

$$\begin{array}{rl}
Q = & \{\, q_0, q_1, q_2\,\} \\
\Sigma = & \{\, 0, 1\,\} \\
F = & \{\, q_2\,\} \\
\Delta = & \{\, (q_0,1,q_0), (q_0,1,q_1), (q_0,1,q_2), \\
 & (q_0,0,q_0), (q_1,0,q_1), (q_1,0,q_2)\,\}.
\end{array}$$

Another counter example was also provided independently by Kupke [19].

This (strict tree) property on the computation tree of nondeterministic finite automata is found to be satisfied by some automata which are different from the class of deterministic finite automata (**DFA**). In [3], we introduced a class of nfa's between **DFA** and **UNFA**, and we call it **SUFA.** **SUFA** contains properly **DFA** and is properly contained in **UNFA.** Thus we have

$$\textbf{DFA} \subset \textbf{SUFA} \subset \textbf{UNFA}$$

Realized that **SUFA** is the class of automata that satisfy the strict tree property and as far as its asymptotic behavior, they belong to **UNFA.**

As far as the descriptional complexity results on these ambiguity hierarchy of finite automata, the exponential gaps between these classes seems inevitable. This can be attributed to the subset construction used in [26].

For a given sequence of automata $(\mathcal{M}_i)_{i\geq 0}$, we define $|Q_{\mathcal{M}}|(i)$ as the number of states of $\mathcal{M}$. For a function f on the set of natural numbers with $f \notin O(n)$ and two classes X_1 and X_2 of finite automata,we will say that there is a $f(n)$*-size gap* or $f(n)$ *separation* between X_1 and X_2, denoted by

$$X_1 \prec_{f(n)} X_2,$$

if there is a sequence of regular languages $L_i \in (2^{\Sigma^*})^N$ for $i \geq 0$, such that there exists a sequence $(\mathcal{M}_i)_{i\geq 0} \in X_2^N$ with $L(\mathcal{M}_i) = L_i$ and $|Q_{\mathcal{M}}| \in O(i)$ and all sequences $(\mathcal{N}_i)_{i\geq 0} \in X_1^N$ with $L(\mathcal{N}_i) = L_i$ satisfy $|Q_{\mathcal{N}}| \in \Omega(f(i))$.

The following gaps between ambiguity classes have been established:

$$\textbf{DFA} \prec_{2^n} \textbf{UNFA} \prec_{2^n} \textbf{CAFA}$$

and

$$\textbf{PAFA} \prec_{2^n} \textbf{NFA}.$$

The exponential gap between $s(L)$ and the size of the minimal unambiguous finite automata for some languages L is known since 1978 [29, 27, 32]. The exponential gap between **UNFA** and **CAFA** had been established in [27, 32]. Leung in [21] proved the exponential gap between **PAFA** and **NFA.**

In [3], the exponential gap between **DFA** and **SUFA** is easily obtained, i.e.

$$\textbf{DFA} \prec_{2^n} \textbf{SUFA}.$$

One may regard as a central open question in the context of automata ambiguity the following :

Problem 3 *Prove or disprove the existence of an exponential gap between* **CAFA** *and* **PAFA.**

A partial solution to this problem has been provided in [16], where an exponential gap between the sizes of k-ambiguous nfa's and polynomially ambiguous nfa's was established for every k.

Similar question can be asked between **SUFA** and **UNFA.**

Problem 4 *Prove or disprove the existence of an exponential gap between* **SUFA** *and* **UNFA.**

4. 2DFA AND 2NFA

Classic result in [26] tells us that for any regular language L, we can construct a (minimal) deterministic finite automaton (dfa) M for L with at most $2^{|Q_N|}$ number of finite states from a given (minimal) nondeterministic finite automaton (nfa) N with $|Q_N|$ number of finite states recognizing L.

In particular, suppose we have a sequence of regular languages $\{L_k\}_{k\geq 1}$, where

$$L_k = \{\, u1v \mid u \in \{0,1\}^*, v \in \{0,1\}^{k-1} \,\}$$

for every natural number k. It is easy to realize a minimal nfa with $k+1$ number of states for L_k and a minimal dfa can be constructed with 2^k number of finite states [25, 9].

For the simple generalization of finite automata, which we will call the two-way (non)deterministic finite automata (2dfa, 2nfa), there is no result yet (similar to the basic models) that would fixed the relationship between 2nfa's and 2dfa's.

It is well known that these models of computations accept exactly regular languages [25, 9].

Sakoda and Sipser [28] proposed the following question since 1978:

Let L be any regular language. We denote by $s_2(L)$ the size of the minimal 2dfa accepting strings in L and by $ns_2(L)$ the size of the minimal 2nfa accepting all strings in L.

Problem 5: *Does there exist a polynomial* f*, such that*

$$ns_2(L) \leq f(s_2(L))$$

for every regular language L.

The first attempt to answer this question was done in [28] showing the exponential gap between 2nfa and 2dfa for special automata which allow to read the input several times from left to right.

In 1980, Sipser [31] consider a so-called sweeping automata whose reading head may change direction only at the endmarkers. In [31], it was shown that for a specific sequence of regular languages, namely $\{B_n\}_{n\geq 1}$,

$$ns(B_n) = O(n) \text{ and } s_2(B_n) \geq 2^n,$$

for every natural number n and where $ns(B_n)$ is the size of the minimal nfa for B_n.

The drawback of this result was on the size of B_n which is 2^{n^2}. Obviously, the size of the alphabet of B_n grows with n. Leung [20] proved a maximal possible exponential gap between nondeterminism and determinism in the sweeping automata model for a sequence of regular languages over $\{0,1\}$.

In [13], the idea of degree of non-obliviousness of a 2dfa $\mathcal{M}$ as a function $f_{\mathcal{M}} : N \longrightarrow N$, where $f_{\mathcal{M}}(n)$ is the number of different orders of the indexes of the tape cells appearing in computations of $\mathcal{M}$ on inputs of length n was introduced. It was proved that there is an exponential gap between 1nfa's and 2dfa's with the degree of non-obliviousness bounded by $o(n)$.

Note that Micali [23] proved that deterministic sweeping automata may require a number of states that is exponential in $s_2(L)$ for some specific regular language L. This implies that the previous results do not solve Problem 5.

Unfortunately, Problem 5 remains open until now. This problem became one of the fundamental challenges in the boundary between automata theory and complexity theory. Berman [4] and Sipser [31] showed that if one proves an exponential gap between 2nfa and 2dfa such that the words involved in the proof are polynomial length, then the deterministic logarithmic space (DLOG) is not equal to the nondeterministic logarithmic space (NLOG). Thus Problem 5 is related to the famous open question, DLOG $\subset$ NLOG?

Another way to solve Problem 5 is to prove an existence of a particular regular language witnessing at least a large polynomial gap between the sizes of minimal 2dfa's and minimal 2nfa's. The largest known gap so far is quadratic [14]

Problem 6: *Is there a sequence of regular languages* $\{L_n\}_{n=1}^{\infty}$ *such that*

$$s_2(L_n) \geq f(ns_2(L_n)),$$

where f *is a increasing function that grows asymptotically faster than* n^2.

The language presented by Sipser in [31] is a probable candidate for proving the gap between 2dfa's and 2nfa's.

5. FINAL REMARKS

We have presented some problems which are in the core of automata theory. Certainly this is just some of the many still hard open problems in the area that need to be solved. Moreover, there also several open problems about probabilistic and quantum finite automata or automata accepting infinite words. Working on these problems will help automata theory increases it popularity and its acceptance to theoretical computer science community. Progress on the methods of solving these problems is possible.

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